Sine Wave with Nonzero Baseline

The objective of this case study is to investigate how hourly tidal water level data oscillates over time during a short continuous observation window and to quantify that relationship using nonlinear curve fitting in **Isalos Analytics Platform**. In this case study, the **Sine wave with nonzero baseline** model is used to estimate the oscillation amplitude, wavelength, phase shift, and the constant vertical offset around which the water level varies. The dataset used in this case study is derived from **NOAA CO-OPS tide-gauge water level observations**.

The dataset was arranged so that the independent variable (X) is a **numeric time index**, such as the number of hours since the first selected observation, and the dependent variable (Y) is the measured water level in consistent units relative to the selected datum. Before importing into Isalos, the timestamp column should be converted into a linear numeric scale, because the model expects X to be on a linear scale.

The **Sine wave with nonzero baseline model** describes periodic oscillations around a constant offset rather than around zero. The equation used in this analysis is:

$$Y = Amplitude \cdot \sin\left(\left(\frac{2\pi X}{Wavelength}\right) + PhaseShift\right) + Baseline$$

In this model, **Amplitude** represents the peak value of the oscillation, **Wavelength** represents the period of the repeating cycle expressed in X units, **PhaseShift** represents the horizontal offset in radians, and **Baseline** represents the constant vertical offset about which the signal oscillates.

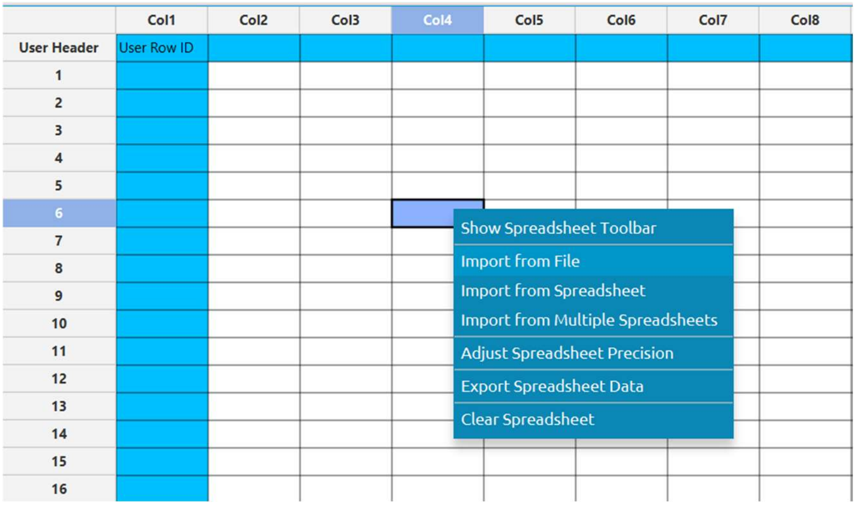
The purpose of this analysis is to determine whether the observed tidal data during the selected interval are consistent with a sinusoidal pattern around a nonzero baseline and to convert the raw water-level measurements into meaningful quantitative outputs. The main results obtained from the fit are **Amplitude**, **Wavelength**, **PhaseShift**, and **Baseline**. These results are useful because they allow quantitative characterization of the tidal oscillation and support interpretation of the periodic structure of the water-level signal.

Isalos version used: 2.0.2

Scientific article: https://rmets.onlinelibrary.wiley.com/doi/10.1002/gdj3.42

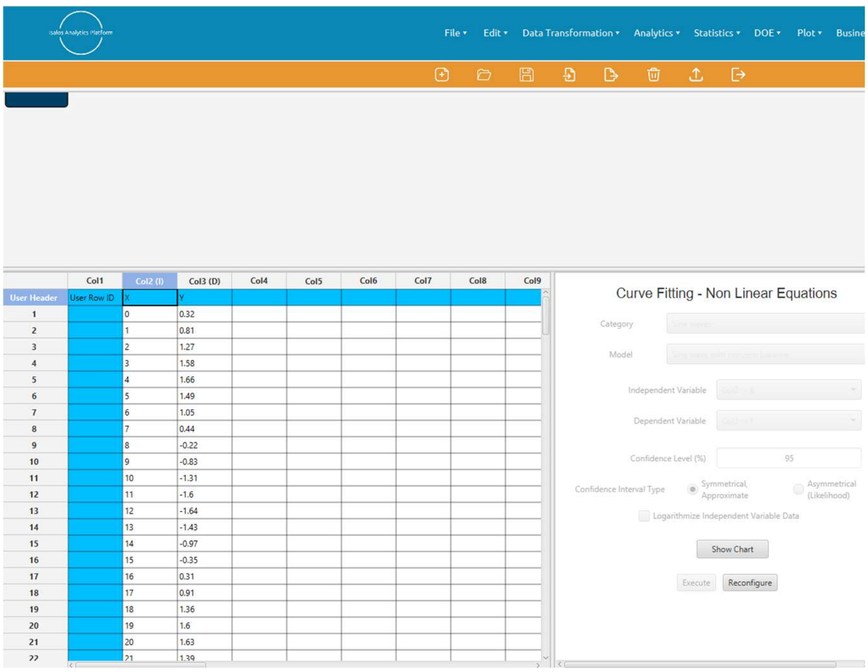
Step 1: Import data from file

Right-click on the input spreadsheet panel on the left and choose “**Import from File”**. Then browse to the file containing the XY dataset for this case study and load the sheet in which each row corresponds to one hourly time value and one water-level value.



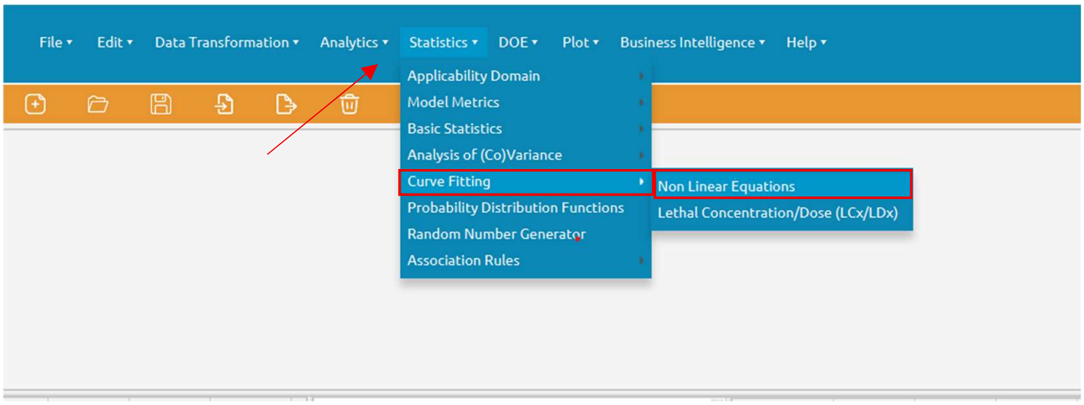
**(Before importing into Isalos, convert the timestamp column into a numeric time column so that the first selected observation becomes Hour 0, the next becomes Hour 1, and so on. This is necessary because the Sine wave with nonzero baseline model requires a linear X variable.)*

The data will appear on the left spreadsheet.

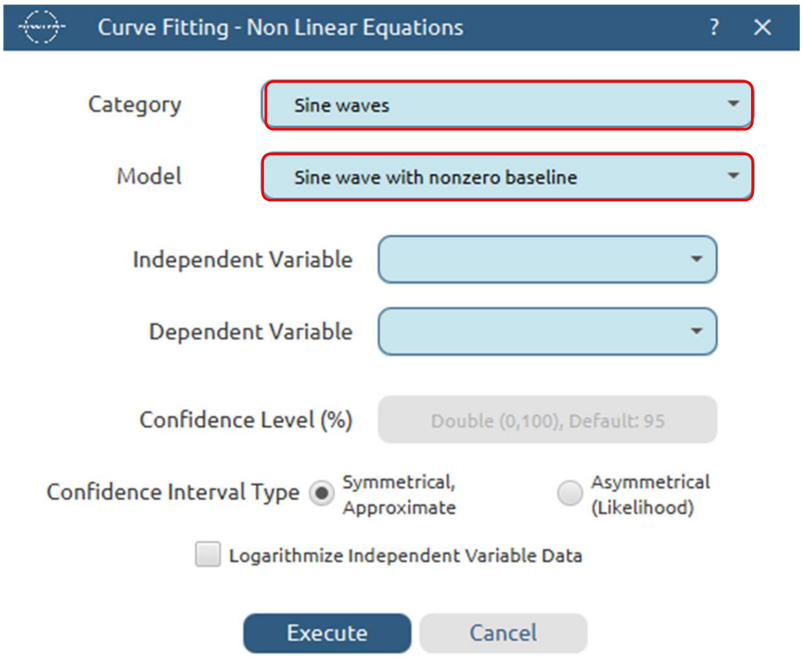


Step 2: Select the desired statistical analysis model

From the toolbar, open the Statistics drop-down list and navigate through: **Statistics > Curve Fitting > Non Linear Equations**.

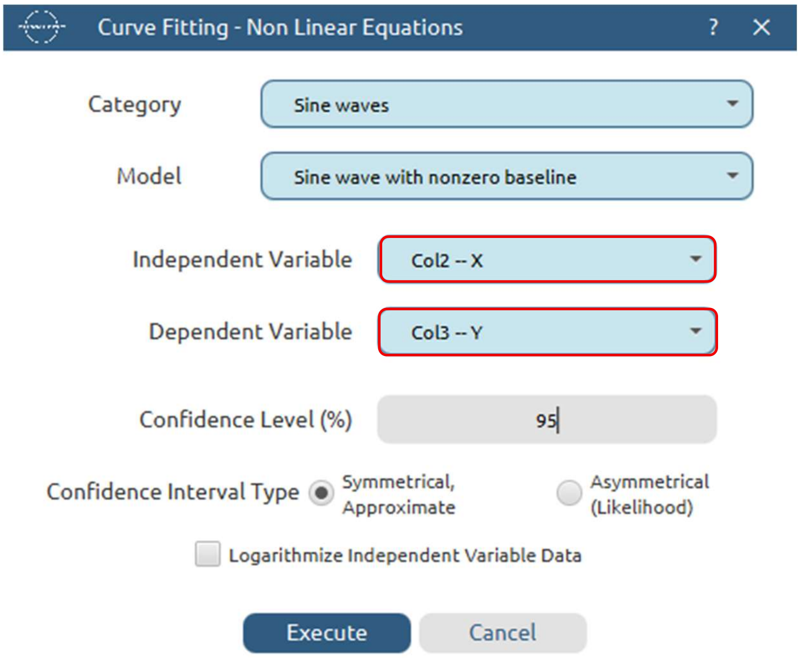


In the category list, select the appropriate category containing **sine wave** equations and then choose the **Sine wave with nonzero baseline** model from the model menu.



Step 3: Configure variables and confidence intervals

Set the column containing the hour values as the independent variable (X), and set the column containing the water-level values as the dependent variable (Y). Set the confidence level to **95%** and choose **Symmetrical Approximate** as the confidence interval type.

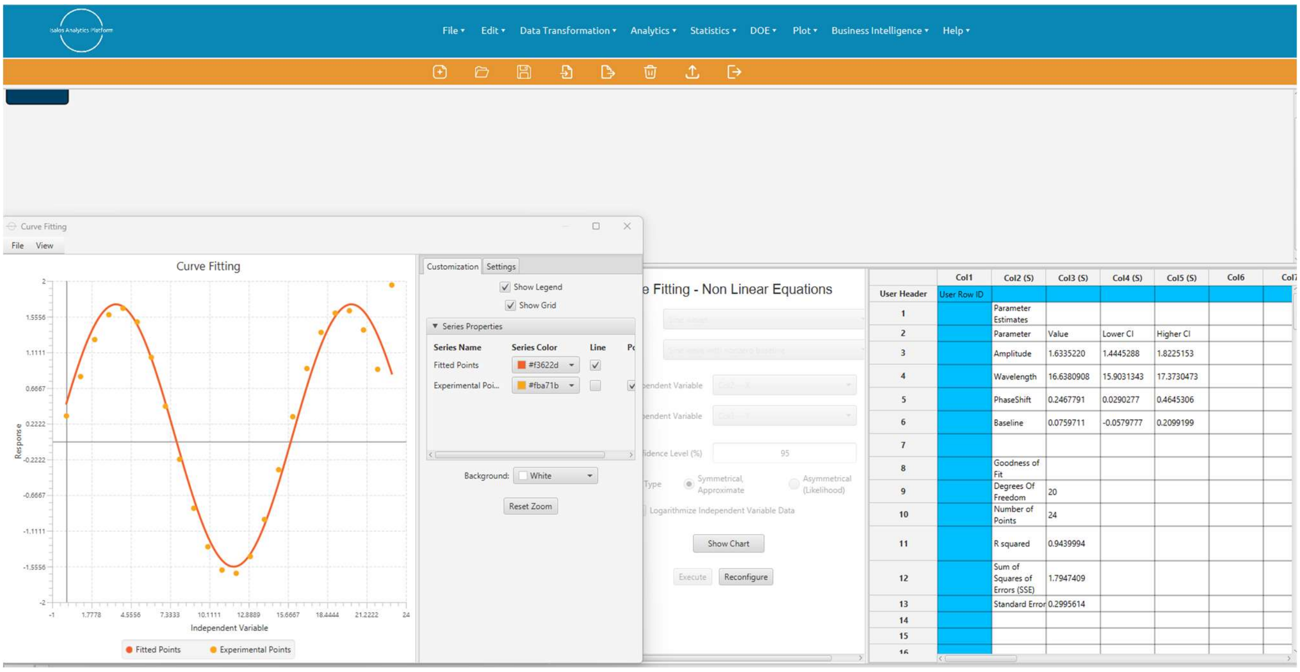


Step 4: Analyze the output and fitted curve

Once the analysis is completed, Isalos presents the fitted sine curve together with the experimental data points, allowing direct visual assessment of how well the model captures the oscillatory pattern in the water-level data. In this study, the graph shows a repeating wave pattern around a steady offset, which is characteristic of tidal variation over a short interval.

The results page presents the estimated model parameters with their confidence limits, together with the corresponding fitted plot. In the Sine wave with nonzero baseline model, the main fitted parameters are **Amplitude**, **Wavelength**, **PhaseShift**, and **Baseline**, and these values collectively determine the height, period, timing, and vertical offset of the fitted oscillation.

The **Goodness of Fit** section of the table summarizes key fitting statistics such as **the number of data points** used, **degrees of freedom**, **residual sum of squares**, and the **standard error** of the regression. These outputs should be interpreted together with the fitted plot in order to evaluate how well the model describes the tidal oscillation and how reliable the estimated parameters are.



The fitted results indicate that the tidal water-level data are well described by a **Sine wave with nonzero baseline** model, showing that the variation in water level over time follows a periodic oscillatory pattern around a constant offset.

The estimated **Amplitude of 1.6335** represents the magnitude of the oscillation from the central baseline to the peak of the wave, the estimated **Wavelength of 16.6381** represents the period of the oscillation expressed in X units (hours), and the estimated **PhaseShift of 0.2468** represents the horizontal displacement of the wave along the time axis. The estimated **Baseline of 0.0760** represents the vertical offset around which the tidal oscillation occurs, indicating that the signal is not centered at zero but fluctuates around a slightly positive mean level.

The confidence intervals for all four parameters are relatively narrow, indicating the fitted model's stability and reliability. The **Amplitude** and **Wavelength** are well constrained, showing a clear and consistent oscillation, while the **PhaseShift** is estimated with reasonable precision. The **Baseline** remains close to zero, confirming a small but consistent vertical offset of the data.

Together with the high **R^2 value of 0.9440** and the relatively low residual error (**SSE = 1.7947**, **Standard Error = 0.2996**), the results indicate that the **Sine wave with nonzero baseline model** provides a good description of the tidal data and that the fitted parameters are reliable for characterizing the amplitude, periodicity, phase, and baseline level of the oscillatory signal in this study.

References:

(1) Woodworth, P.L., Hunter, J.R., Marcos, M., Caldwell, P., Menéndez, M. and Haigh, I., 2016. Towards a global higher-frequency sea level dataset. *Geoscience Data Journal*, *3*(2), pp.50-59.

(2) **National Ocean ServiceNational Oceanic and Atmospheric Administration** https://oceanservice.noaa.gov/annualreport/2020/coops.html